Name: Sahil T Chaudhary
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MCT Project - 1

Exercise 1.

Lateral dynamics :-

$$u = \begin{bmatrix} \delta \\ F \end{bmatrix}, \quad s_1 = \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix}$$

$$\dot{s}_1 = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix}$$

$$= \begin{bmatrix} \dot{y} \\ -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \\ \dot{\psi} \\ \frac{2l_f C_\alpha}{I_z}\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{2l_r C_\alpha}{I_z}\left(-\frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \end{bmatrix}$$

$$A_1 = \frac{\partial f}{\partial s_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{\partial f_2}{\partial \dot{y}} & 0 & \frac{\partial f_2}{\partial \dot{\psi}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{\partial f_4}{\partial \dot{y}} & 0 & \frac{\partial f_4}{\partial \dot{\psi}} \end{bmatrix}$$

where,

$$\frac{\partial f_2}{\partial \dot{y}} = \frac{2C_\alpha}{m}\left(\frac{-\cos\delta}{\dot{x}} - \frac{1}{\dot{x}}\right)$$

$$\frac{\partial f_2}{\partial \dot{\psi}} = -\dot{x} + \frac{2C_\alpha}{m}\left(\frac{-l_f \cos\delta}{\dot{x}} + \frac{l_r}{\dot{x}}\right)$$

$$\frac{\partial f_4}{\partial \dot{y}} = \frac{2 l_f C_\alpha}{I_z}\left(-\frac{1}{\dot{x}}\right) - \frac{2 l_r C_\alpha}{I_z}\left(-\frac{1}{\dot{x}}\right)$$

$$\frac{\partial f_4}{\partial \dot{\psi}} = \frac{2 l_f C_\alpha}{I_z}\left(\frac{-l_f}{\dot{x}}\right) - \frac{2 l_r C_\alpha}{I_z}\left(\frac{l_r}{\dot{x}}\right)$$

$$\therefore A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{2C_\alpha}{m}\left(\frac{-\cos\delta - 1}{\dot{x}}\right) & 0 & -\dot{x} + \frac{2C_\alpha}{m}\left(\frac{l_r - l_f\cos\delta}{\dot{x}}\right) \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2C_\alpha}{\dot{x}I_z}(l_f - l_r) & 0 & -\frac{2C_\alpha}{\dot{x}I_z}(l_r^2 + l_f^2) \end{bmatrix}$$

$$B_1 = \left(\frac{df}{du}\right) = \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m}\left(-\delta\sin\delta + \cos\delta + \sin\delta\left(\frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right)\right) & 0 \\ 0 & 0 \\ \frac{2l_f C_\alpha}{I_z} & 0 \end{bmatrix}$$

$$\therefore \dot{s}_1 = A_1 s_1 + B_1 u$$

Longitudinal dynamics :-

$$u = \begin{bmatrix} \delta \\ F \end{bmatrix}, \quad s_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$\dot{s}_2 = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}$$

$$\dot{s}_2 = \begin{bmatrix} \dot{x} \\ \dot{\psi}\dot{y} + \frac{1}{m}(F - fmg) \end{bmatrix}$$

$$= \begin{bmatrix} \dot{x} \\ \frac{1}{m}(F - fmg) \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\psi}\dot{y} \end{bmatrix}$$

Disturbance

$$A_2 = \frac{\partial f}{\partial s_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$B_2 = \frac{\partial f}{\partial u} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix}$$

$$\therefore \dot{s}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} s_2 + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix} u + \begin{bmatrix} 0 \\ \dot{\psi}\dot{y} \end{bmatrix}$$

Disturbance

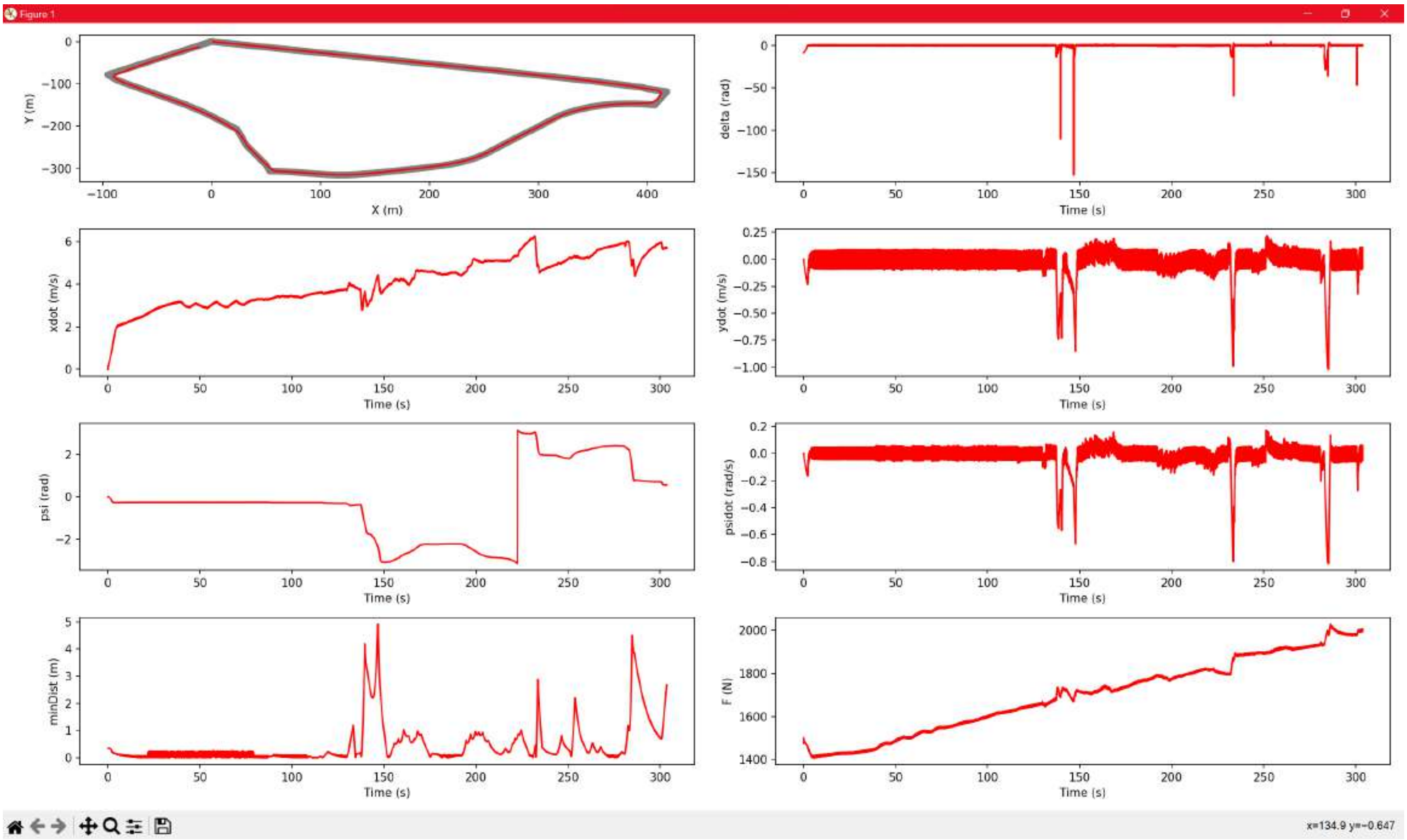

Figure 1
Y (m)
X (m)
delta (rad)
Time (s)
xdot (m/s)
ydot (m/s)
psi (rad)
psidot (rad/s)
minDist (m)
F (N)
x=134.9 y=-0.647

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MCT Project-2

Exercise 1.

1. For lateral control:

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_\alpha}{m\dot{x}} & \frac{4C_\alpha}{m} & \frac{-2C_\alpha(l_f-l_r)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-2C_\alpha(l_f-l_r)}{I_z\dot{x}} & \frac{2C_\alpha(l_f-l_r)}{I_z} & \frac{-2C_\alpha(l_f^2+l_r^2)}{I_z\dot{x}} \end{bmatrix}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \frac{2C_\alpha l_f}{I_z} & 0 \end{bmatrix}\begin{bmatrix} \delta \\ F \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} + [0]u$$

Now, for controllability

$$P = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix}$$

Using MATLAB to calculate,

a) at $\dot{x} = 2\,m/s$

rank(P) $= 4 \Rightarrow$ full rank

$\therefore$ controllable

b) at $\dot{x} = 5\,m/s$

rank(P) $= 4 \Rightarrow$ full rank

$\therefore$ controllable

c) at $\dot{x} = 8\,m/s$

rank(P) $= 4 \Rightarrow$ full rank

$\therefore$ controllable

Now, for observability

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}$$

Using MATLAB to calculate,

a) at $\dot{x} = 2\,m/s$

$\text{rank}(O) = 4 \Rightarrow$ full rank

$\therefore$ observable

b) at $\dot{x} = 5\,m/s$

$\text{rank}(O) = 4 \Rightarrow$ full rank

$\therefore$ observable

c) at $\dot{x} = 8\,m/s$

$\text{rank}(O) = 4 \Rightarrow$ full rank

$\therefore$ observable

For longitudinal control :-

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix}\begin{bmatrix} \delta \\ F \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} + [0]\, u$$

Now, for controllability

$$P = [B \quad AB]$$

$\text{rank}(P) = 2 \Rightarrow$ full rank

$\therefore$ controllable

And for observability

$$O = \begin{bmatrix} C \\ CA \end{bmatrix}$$

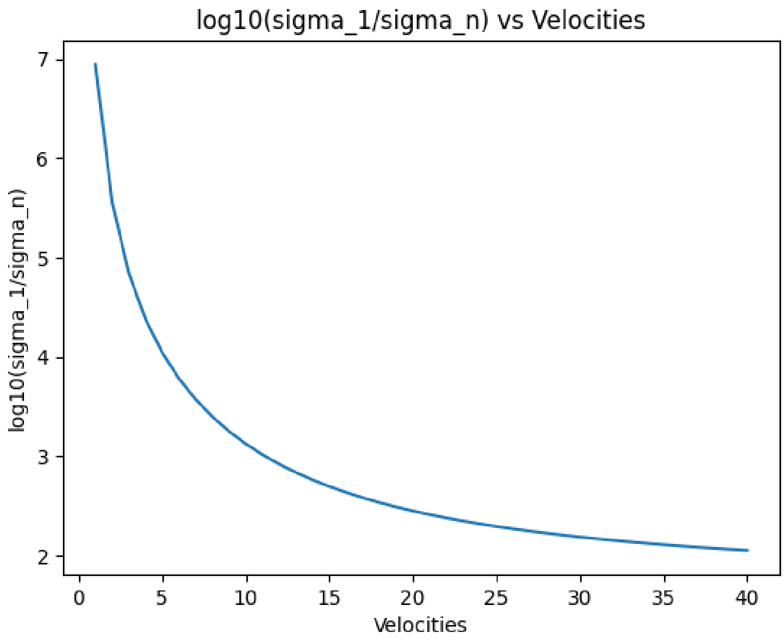
$\text{rank}(O) = 2 \Rightarrow$ full rank

$\therefore$ observable.

The longitudinal controller's controllability & observability don't depend on $\dot{x}$, as $\dot{x}$ is a state variable.

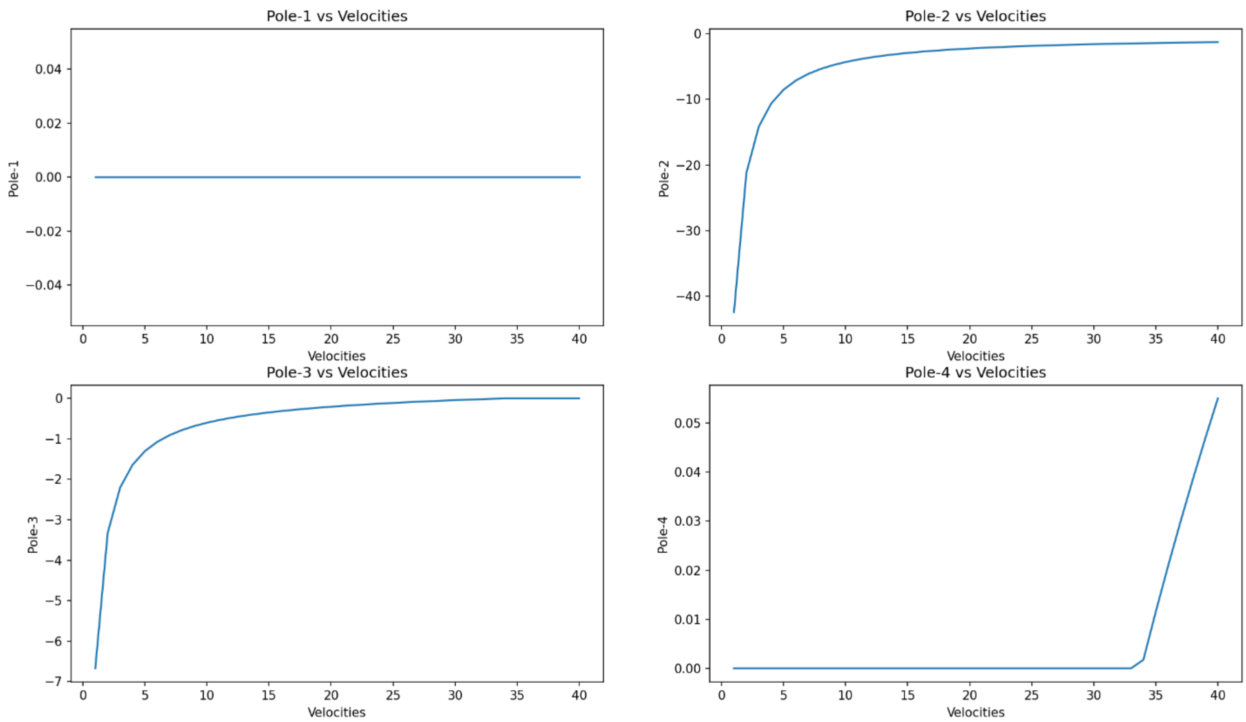
Exercise 1:

Q2. (a)



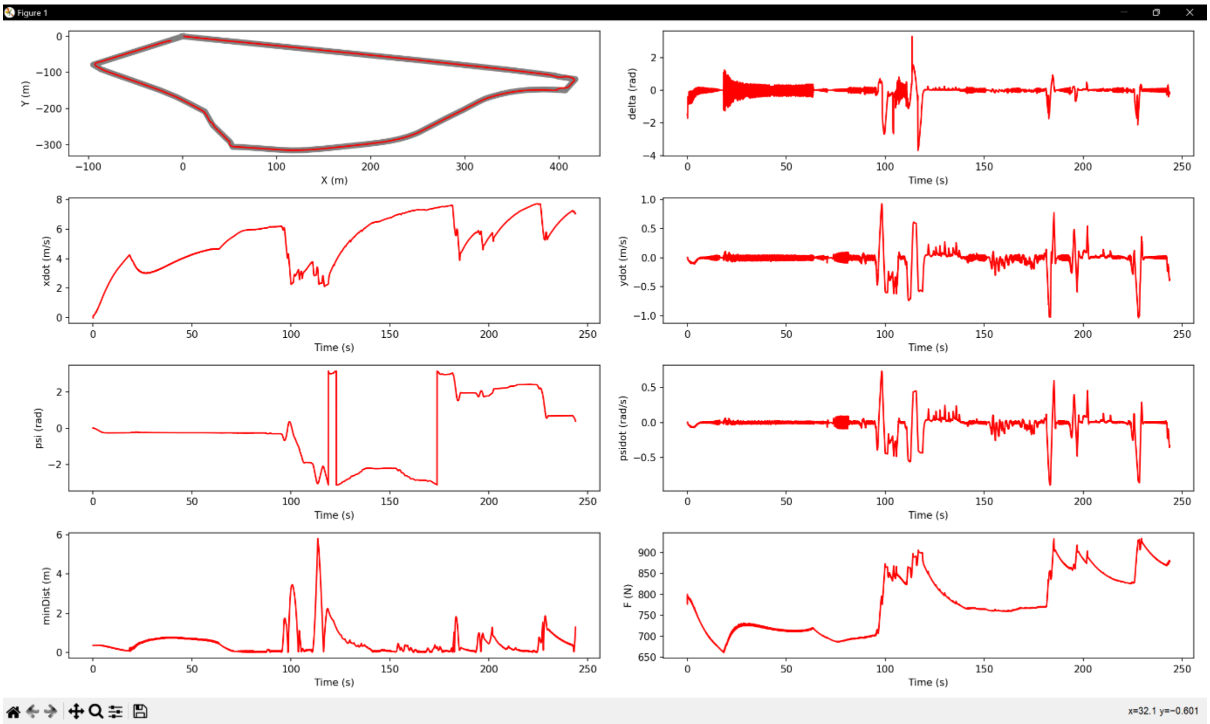
Since the ratio of sigma_1 and sigma_n is decreasing with increasing velocity, it implies that the singular value corresponding to the least controllable state is increasing. Hence, the controllability increases.

Q2. (b)



Apart from Pole-1 (which is constant at the origin), the other poles increase in value and go towards zero. Hence, the stability of the system decreases with increasing velocity.

Exercise 2:




```
Score for average distance: 30.0/30.0
Score for maximum distance: 30.0/30.0
Your time is  243.776
Your total score is : 100.0/100.0
total steps:  243776
maxMinDist:  5.804970845530392
avgMinDist:  0.5007847578327572
INFO: 'main' controller exited successfully.
```

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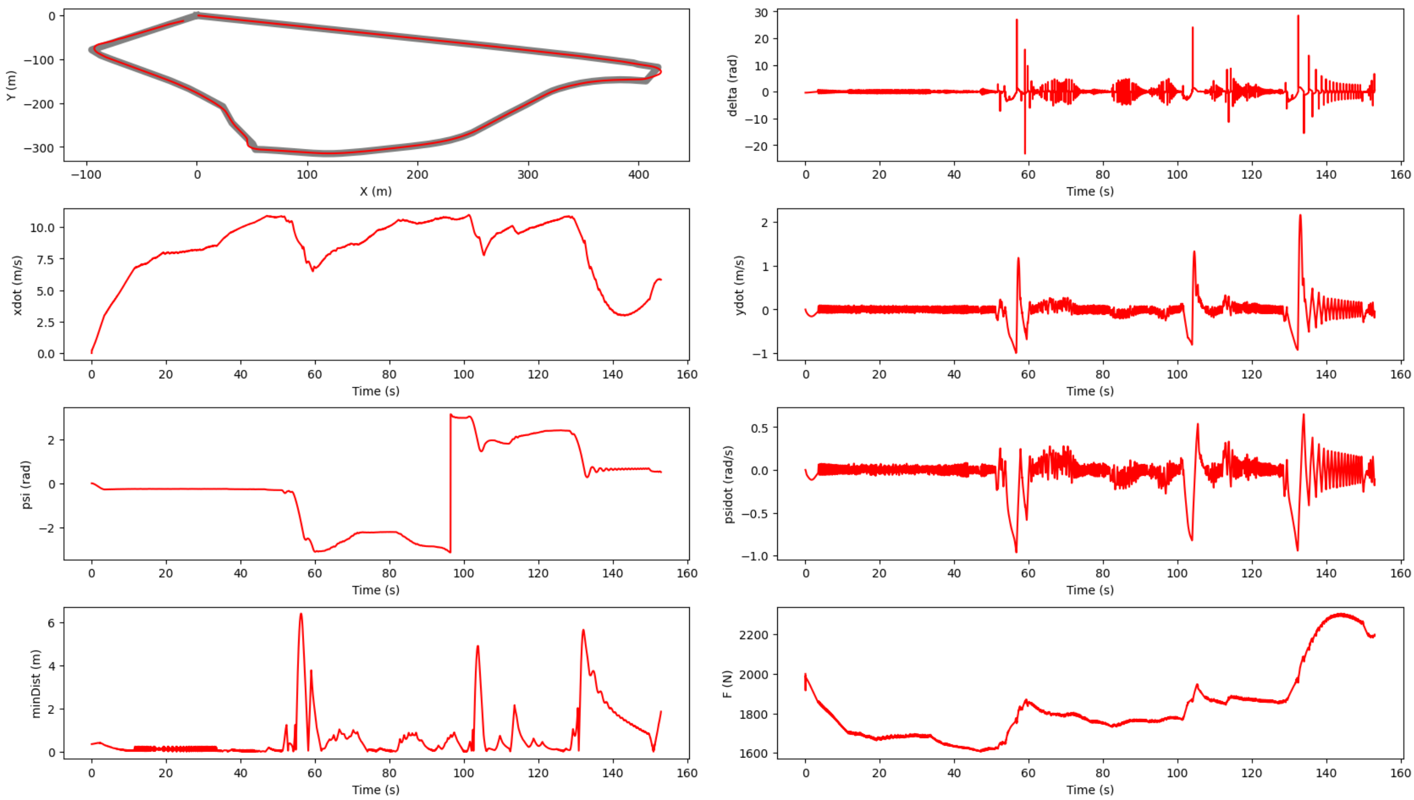
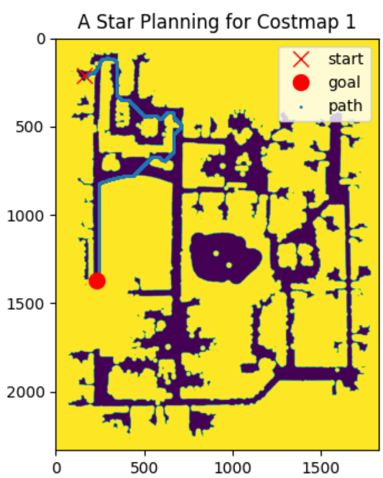
PROJECT-3

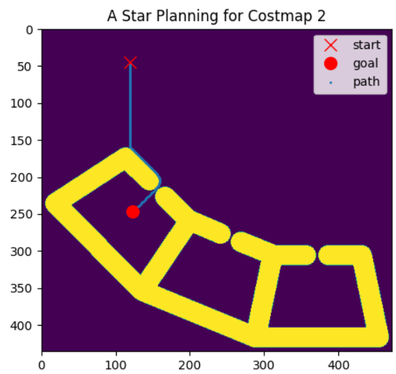
Exercise 1.

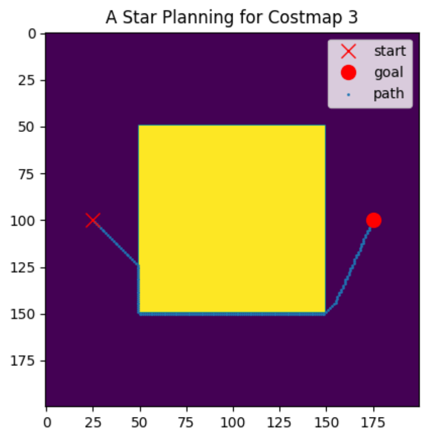


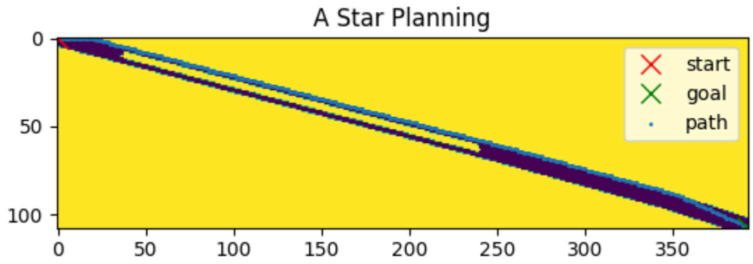

```
Console - All
Score for average distance: 30.0/30.0
Score for maximum distance: 30.0/30.0
Your time is  152.96
Your total score is : 100.0/100.0
total steps:  152960
maxMinDist:  6.394890980743181
avgMinDist:  0.696712017998461
INFO: 'main' controller exited successfully.
```

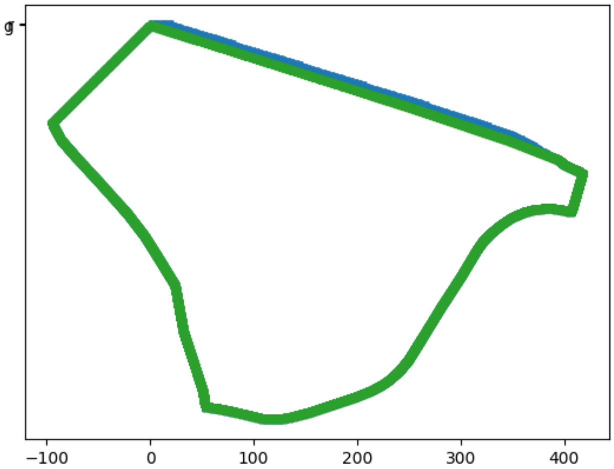
Exercise 2.

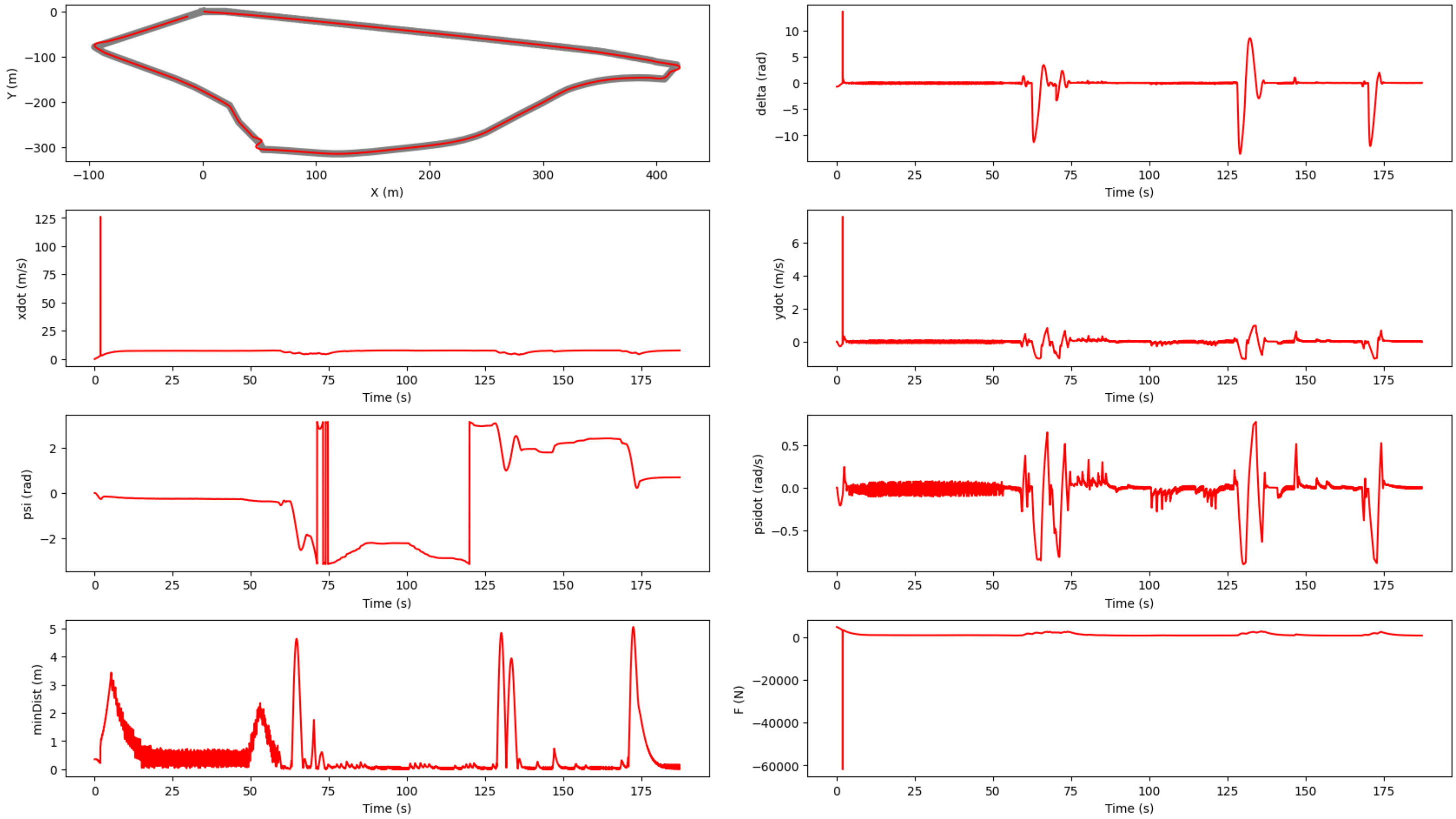












Time of completion (based on graphs above): 190s

```
Console - All
INFO: main: Starting controller: C:\Users\sahil\AppData\Local\Programs\Python\Python38\python.exe -u main.py
INFO: obstacle_controller: Starting controller: C:\Users\sahil\AppData\Local\Programs\Python\Python38\python.exe -u obstacle_controller.py
map size (108, 393)
reach goal
path length 393
total steps:  187296
maxMinDist:  5.04459945001405
avgMinDist:  0.5516731107102213
INFO: 'main' controller exited successfully.
```

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MCT Project -4

Exercise 1.

Process Model :-

$$X_{t+1} = X_t + \delta t(\dot{x}_t \cos\psi_t - \dot{y}_t \sin\psi_t) + w_t^x$$
$$Y_{t+1} = Y_t + \delta t(\dot{x}_t \sin\psi_t + \dot{y}_t \cos\psi_t) + w_t^y$$
$$\psi_{t+1} = \psi_t + \delta t\, \dot{\psi}_t + w_t^\psi$$

$$\therefore x_{t+1} = f(x_t, u_t) + w_t$$

$$\therefore f = x_t + \delta t\, \dot{x}_t$$

$$= \begin{bmatrix} X_t \\ Y_t \\ \psi_t \\ m_x^1 \\ m_y^1 \\ \vdots \\ m_x^n \\ m_y^n \end{bmatrix} + \delta t \begin{bmatrix} \dot{X}_t \\ \dot{Y}_t \\ \dot{\psi}_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$f = \begin{bmatrix} x_t + \delta_t(\dot{x}_t \cos\varphi_t - \dot{y}_t \sin\varphi_t) \\ y_t + \delta_t(\dot{x}_t \sin\varphi_t + \dot{y}_t \cos\varphi_t) \\ \varphi_t + \delta t\, \dot{\varphi}_t \\ m_x^1 \\ m_y^1 \\ \vdots \\ m_x^n \\ m_y^n \end{bmatrix}$$

$$\therefore F = \frac{df}{dx}$$

$$F_t = \begin{array}{c} \left[\begin{array}{ccccccc} 1 & 0 & -\delta t(\dot{x}_t \sin\varphi_t + \dot{y}_t \cos\varphi_t) & 0 & \cdots & & 0 \\ 0 & 1 & \delta t(\dot{x}_t \cos\varphi_t - \dot{y}_t \sin\varphi_t) & 0 & \cdots & & 0 \\ 0 & 0 & 1 & 0 & \cdots & & 0 \\ 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & & & \\ 0 & 0 & 0 & 0 & \cdots & & 1 \end{array}\right] \begin{array}{l} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} 3 \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} 2n \end{array} \\ \underbrace{\qquad\qquad\qquad\qquad}_{3} \quad \underbrace{\qquad\qquad\qquad\qquad}_{2n} \end{array}$$

Measurement Model :-

$$y_t = \begin{bmatrix} \left[(m^1_x - x_t)^2 + (m^1_y - y_t)^2\right]^{1/2} \\ \left[(m^2_x - x_t)^2 + (m^2_y - y_t)^2\right]^{1/2} \\ \vdots \\ \operatorname{atan2}(m^1_y - y_t,\, m^1_x - x_t) - \psi_t \\ \operatorname{atan2}(m^2_y - y_t,\, m^2_x - x_t) - \psi_t \end{bmatrix}_{2n \times 1} + v_t$$

$\downarrow$

$h(x_t)$

$$\therefore H = \frac{\partial h}{\partial x}$$

$$H = \begin{array}{c} \\ n\left\{ \vphantom{\begin{array}{c}a\\a\\a\\a\\a\\a\end{array}} \right. \\ n\left\{ \vphantom{\begin{array}{c}a\\a\\a\\a\end{array}} \right. \end{array} \begin{array}{c} \begin{array}{cc} \overbrace{\hspace{14em}}^{3} & \overbrace{\hspace{18em}}^{2n} \end{array} \\ \left[\begin{array}{ccccccccc} \frac{-(m^1_x - x_t)}{\|m^1 - p_t\|} & \frac{-(m^1_y - y_t)}{\|m^1 - p_t\|} & 0 & \frac{m^1_x - x_t}{\|m^1 - p_t\|} & \frac{m^1_y - y_t}{\|m^1 - p_t\|} & 0 \cdots 0 & & & \\ \frac{-(m^2_x - x_t)}{\|m^2 - p_t\|} & \frac{-(m^2_y - y_t)}{\|m^2 - p_t\|} & 0 & 0 & 0 & \frac{m^2_x - x_t}{\|m^2 - p_t\|} & \frac{m^2_y - y_t}{\|m^2 - p_t\|} & 0 \cdots 0 & \\ \vdots & \vdots & \vdots & & & \vdots & \vdots & & \\ \frac{m^1_y - y_t}{\|m^1 - p_t\|^2} & \frac{x_t - m^1_x}{\|m^1 - p_t\|^2} & -1 & \frac{y_t - m^1_y}{\|m^1 - p_t\|^2} & \frac{m^1_x - x_t}{\|m^1 - p_t\|^2} & 0 \cdots 0 & & & \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & & \end{array} \right] \end{array}$$

Rough work :-

$$\tan^{-1}\left[\frac{m_y' - Y_t}{m_x' - X_t}\right]$$

wrt X_t

$$= \frac{m_y - Y_t}{(X_t - m_x)^2\left[\frac{(Y_t - m_y)^2}{(X_t - m_x)^2} + 1\right]}$$

$$= \frac{m_y - Y_t}{(Y_t - m_y)^2 + (X_t - m_x)^2}$$

wrt Y_t

$$= \frac{1}{(X_t - m_x)\left[\frac{(Y_t - m_y)^2}{(X_t - m_x)^2} + 1\right]}$$

$$= \frac{1}{\frac{(X_t - m_y)^2}{X_t - m_x} + X_t - m_x}$$

$$= \frac{\dfrac{1}{\|m' - \mu_t\|^2}}{x_t - m_x}$$

$2(\text{row}) + 3$

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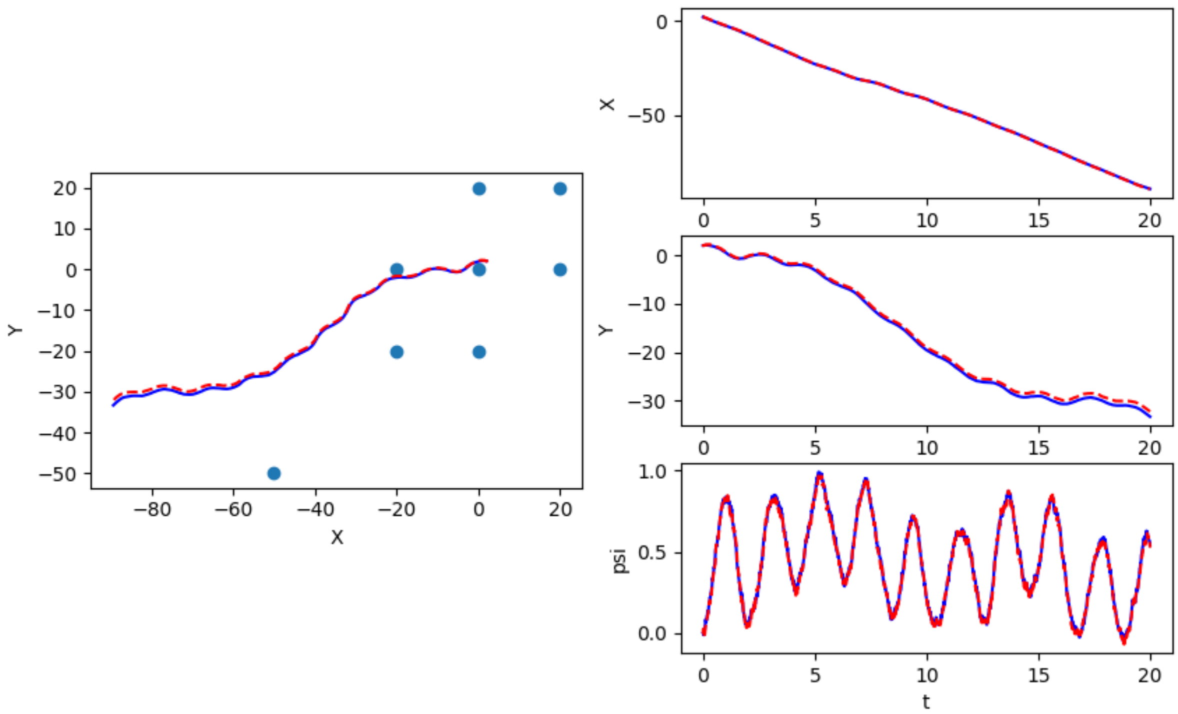
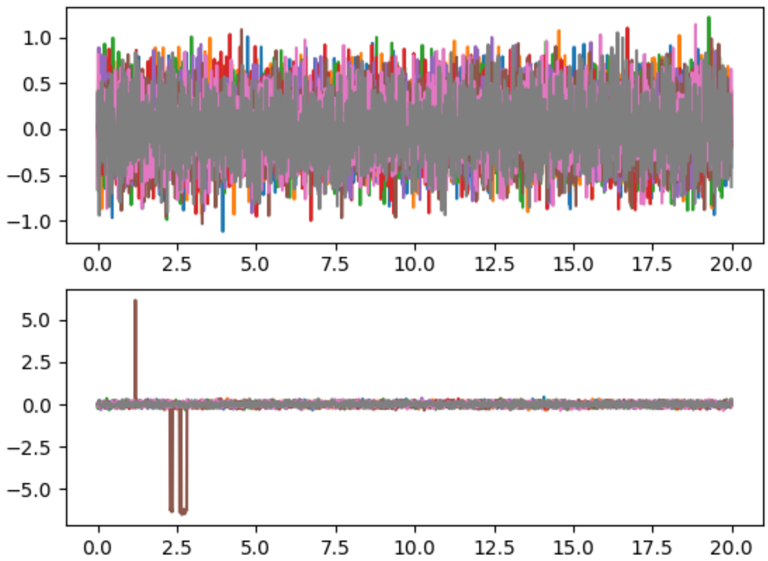
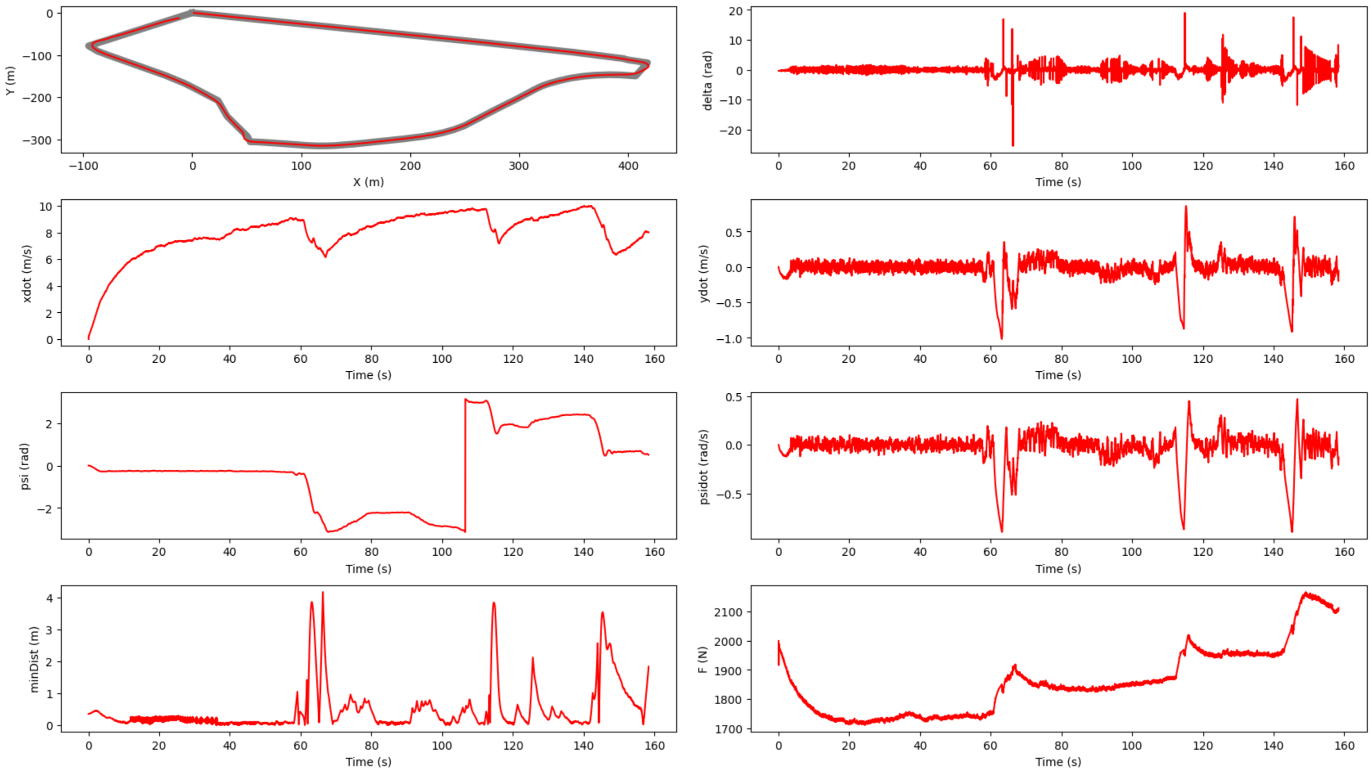
<u>**PROJECT-4**</u>

Exercise 2.








```
Console - All
Score for completing the loop: 30.0/30.0
Score for average distance: 30.0/30.0
Score for maximum distance: 30.0/30.0
Your time is  158.368
Your total score is : 100.0/100.0
total steps:  158368
maxMinDist:  4.171348433108599
avgMinDist:  0.48647716041721717
INFO: 'main' controller exited successfully.
```

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Project - 5

Exercise 1.

$$\dot{x}_p = A_p x_p + B_p \Delta u$$

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix}}_{12\times1} = \underbrace{\begin{bmatrix} 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&g&0&0&0&0&0&0&0 \\ 0&0&0&-g&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}}_{12\times12} \underbrace{\begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}}_{12\times1}$$

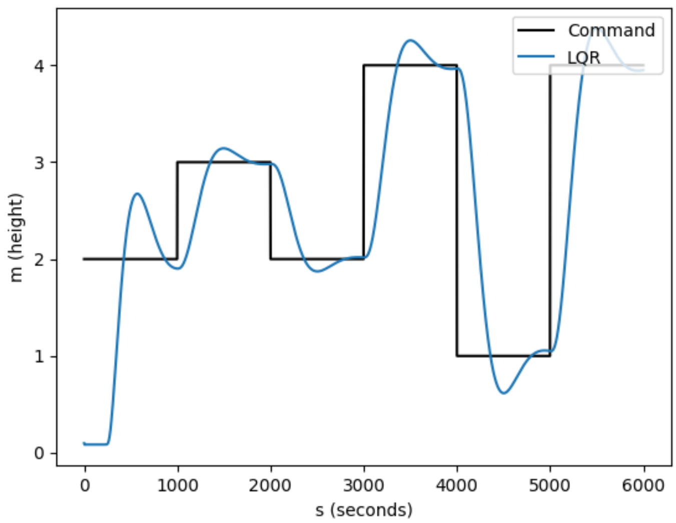
$$+ \underbrace{\begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 1/m&0&0&0 \\ 0&1/I_x&0&0 \\ 0&0&1/I_y&0 \\ 0&0&0&1/I_z \end{bmatrix}}_{12\times4} \underbrace{\begin{bmatrix} \Delta v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}}_{4\times1}$$

$$y_p = C_p x_p$$

$$\underbrace{\begin{bmatrix} x \\ y \\ z \\ \psi \end{bmatrix}}_{4\times 1} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{4\times 12} \underbrace{\begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}}_{12\times 1}$$

PORJECT – 5

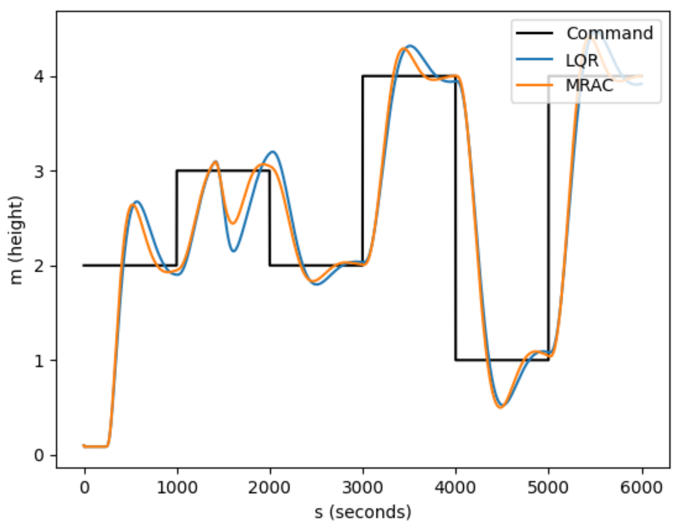
Exercise 1.




```
Console - All
Time:  59.980000000000004
[69.34920542 69.47646216 69.04866806 69.17648112]
Time:  59.99
[69.34864309 69.47589873 69.04810789 69.17591996]
Time:  60.0
===============YOUR RESULT===============
ERROR:  0.204
SCORE:  50.000
INFO:  'ex1_controller'  controller  exited  successfully.
```

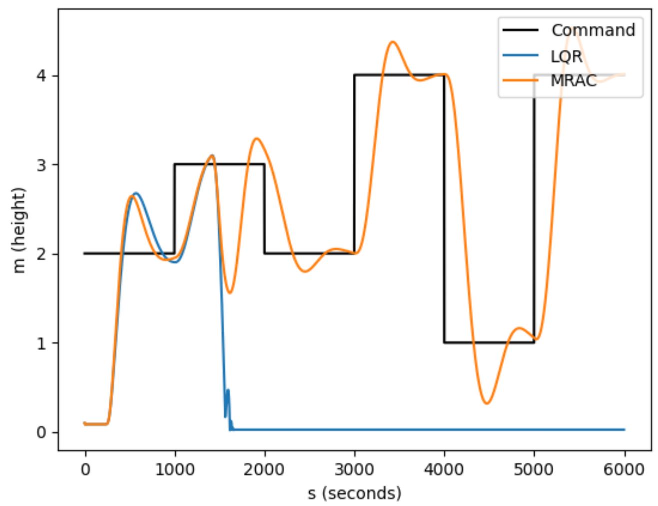
Exercise 2.

1. Loss of thrust = 0.5



```
Console - All
Time: 59.97
--- Motor Failure ---
Time: 59.980000000000004
--- Motor Failure ---
Time: 59.99
--- Motor Failure ---
Time: 60.0
INFO: 'ex2_controller' controller exited successfully.
```

2. Loss of thrust = 0.7




```
Console - All
Time: 59.97
--- Motor Failure ---
Time: 59.980000000000004
--- Motor Failure ---
Time: 59.99
--- Motor Failure ---
Time: 60.0
INFO: 'ex2_controller' controller exited successfully.
```

Even MRAC failed at loss of thrust = 0.8